Calculus Warm Up #9- 2

1. What is a logarithm?

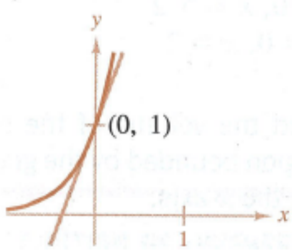
Use properties of logarithms to simplify:

2. $\ln e^{2x}$ 3. $e^{\ln 3x}$ 4. $\ln\left(\frac{3e^2}{e^x}\right)$

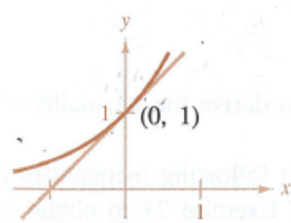
Questions, p. 369

In Exercises 1–6, find the slope of the tangent line to the given exponential function at the point (0, 1).

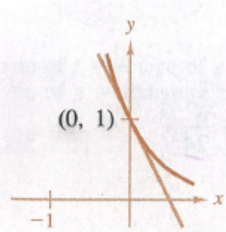
1. $y = e^{3x}$



3. $y = e^x$



5. $y = e^{-2x}$



In Exercises 7–24, find the derivative of the given function.

7. $y = e^{2x}$

9. $y = e^{-2x+x^2}$

11. $f(x) = e^{1/x}$

13. $g(x) = e^{\sqrt{x}}$

15. $f(x) = (x + 1)e^{3x}$

$$f'(x) = (x+1)(e^{3x})(3) + (1)e^{3x}$$
$$= e^{3x}(3x+3+1)$$
$$= e^{3x}(3x+4)$$

17. $f(x) = \dfrac{e^{x^2}}{x}$

19. $y = (e^{-x} + e^{x})^3$

$$f'(x) = \frac{x(e^{x^2})(2x) - (e^{x^2})(1)}{x^2}$$
$$= \frac{e^{x^2}(2x^2-1)}{x^2}$$

21. $f(x) = \dfrac{2}{e^x + e^{-x}}$ $\rightarrow$ $2(e^x + e^{-x})^{-1}$

23. $y = xe^x - e^x$

$f'(x) = -2(e^x + e^{-x})^{-2}(e^x + (e^{-x})(-1))$

$= \dfrac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}$

In Exercises 25 and 26, use implicit differentiation to find dy/dx.

25. $xe^y - 10x + 3y = 0$

$xe^y \frac{dy}{dx} + (1)e^y - 10 + 3\frac{dy}{dx} = 0$

In Exercises 27–30, find the second derivative of the exponential function.

27. $f(x) = 2e^{3x} + 3e^{-2x}$

$f'(x) = 2(e^{3x})(3) + 3(e^{-2x})(-2)$

$= 6e^{3x} - 6e^{-2x}$

$f''(x) = 6(e^{3x})(3) - 6(e^{-2x})(-2)$

$= 18e^{3x} + 12e^{-2x}$

7.2 day 2

-More derivatives of exponential functions

You try, then we'll check in...

Implicit Differentiation

let $u = xy$

$\frac{du}{dx} = x\frac{dy}{dx} + (1)y$

Find $\frac{dy}{dx}$ $\qquad e^{xy} + x^2 - y^2 = 10$

$$\underbrace{(e^{xy})\left(x\frac{dy}{dx} + y\right)} + 2x \underbrace{- 2y\frac{dy}{dx}} = 0$$

$$\underbrace{xe^{xy}\frac{dy}{dx}} + ye^{xy} + 2x \underbrace{- 2y\frac{dy}{dx}} = 0$$

$$\frac{dy}{dx}\left(xe^{xy} - 2y\right) = -2x - ye^{xy}$$

$$\frac{dy}{dx} = -\frac{2x + ye^{xy}}{xe^{xy} - 2y}$$

Find any extrema and points of inflection if they exist, then sketch the graph. Check with your grapher.

Plan

f' & f''

Set $= 0$

test

$$f(x) = \frac{e^x - e^{-x}}{2}$$

$$f(x) = \frac{e^x}{2} - \frac{e^{-x}}{2}$$

$$f'(x) = \frac{e^x}{2} + \frac{e^{-x}}{2} \qquad f'(x) \neq 0$$

$$f''(x) = \frac{e^x}{2} - \frac{e^{-x}}{2} \qquad f''$$

0

$$\frac{e^{-x}}{2} = \frac{e^x}{2}$$

$$x = 0$$

Find the second derivative:

$$f(x) = 5e^{-x} - 2e^{-5x}$$

$$f'(x) = (5e^{-x})(-1) - (2e^{-5x})(-5)$$

$$f'(x) = -5e^{-x} + 10e^{-5x}$$

$$f''(x) = (-5e^{-x})(-1) + (10e^{-5x})(-5)$$

$$f''(x) = 5e^{-x} - 50e^{-5x}$$

HW: p. 369

29 - 35 odd,

then work on AP Rev WS # 4

Answers WS#4

3a) $R'(45) = 1.5\ gal/min^2$

b) $R''(45) = 0$ bc Max of R' @ $t = 45$

4a) $(-3,-2)$ bc f' is positive

b) $x = 0, 2$ where f' goes from increasing to decr. or decr. to incr.

c) $y = -2x + 3$

5a) Show → start with $V = 25\pi h$

then $\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

plug in given stuff $\left(\frac{dV}{dt}\right)$ & isolate $\frac{dh}{dt}$.

Show steps

6a) $\lim_{x\to 3^-} f(x) \stackrel{?}{=} \lim_{x\to 3^+} f(x)$

show they match

yes

c) $\underbrace{2k = 3m + 2}$ Need $\underbrace{2^{nd} \text{ relationship}}$ & solve system

from continuous, so

$k\sqrt{3+1} = m(3) + 2$

derivatives from left & rt =

find $g'(x)$, then

$\lim_{x\to 3^-} g'(x) = \lim_{x\to 3^+} g'(x)$

Solve system → $k = \frac{8}{5}$ $m = \frac{2}{5}$

1. Show $\longrightarrow$ $f(3) = 9$

$f'(3) = -3$

tangent line matches line l

6. $f''(x) = \frac{x^2}{2} + \sqrt{f(x)}$

$f''(3) = \frac{19}{2}$